Noise-aided control of chaotic dynamics in a logistic map

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Controlling chaos involves employing small perturbations to a control parameter, resulting in the stabilization of the system (naturally chaotic) on one of the infinite unstable periodic orbits embedded in the chaotic attractor. In this Brief Report we study the constructive role of external noise in increasing the efficiency of controlling chaos. Using a logistic map as an example, control of chaotic dynamics is achieved using a linear delayed-feedback strategy. Working in the subthreshold regime of control (where the value of control constant is less than the minimum value required to stabilize the period-1 target state), system dynamics in the presence of superimposed noise (system plus control plus noise) exhibit a resonance effect. Furthermore it is observed that the time required to reach the target state decreases appreciably in the presence of an optimum level of noise.

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I. INTRODUCTION

The idea of controlling chaos using a feedback technique was introduced by Ott, Grebogi, and Yorke (OGY) [1]. In this control strategy, small perturbations are applied to a control parameter while monitoring the system in a state space of dynamical variables. When the trajectory crosses a Poincaré section of the chaotic attractor, the size of the perturbation is calculated according to a control formula [1], and an appropriate correction is superimposed on an accessible control parameter for the next cycle. Since there is always an error in targeting the fixed point on the section (corresponding to the desired periodic behavior), the procedure is repeated every cycle. As a result of these perturbations, the system is stabilized on one of the unstable periodic orbits embedded in the chaotic attractor. The OGY algorithm and its subsequent modifications [2–4] have been used to control chaotic dynamics in various experimental systems [5–10].

Another idea that has attracted immense interest lately emanates from the pioneering work of Benzi *et al.* [11]. Since their landmark paper it has been realized that detection of weak signals in nonlinear systems can indeed be improved by increasing the noise level, reaching a maximum signal to noise ratio for an optimum level of noise subsequent to which the signal is masked by noise. This phenomena known as stochastic resonance has been detected and analyzed in a number of physical [12], chemical [13], and biological [14] systems. Moreover it has also been reported that in certain situations superimposed noise can indeed play a constructive role if manipulated judiciously [15].

In this Brief Report the following question is asked: Can external noise if implemented appropriately play a constructive role in the control of chaotic dynamics ? If yes, we seek to verify the existence of the resonance effect reported for various numerical and experimental situations [16,17,12–14]. The paper is organized as follows: In Sec. II we discuss the implementation of delayed-feedback control on the logistic map in the subthreshold regime. In Sec. III, we present numerical results describing the constructive role of noise in the control of chaos and the associated resonance phenomena. Finally, we present a brief discussion regarding the possible relevance of our numerical results to experimental situations.

II. SUBTHRESHOLD CONTROL OF CHAOTIC DYNAMICS

Our model system is a logistic map of the following form:

$$X_{n+1}=aX_n(1-X_n). \tag{1}$$

This discrete system undergoes a standard period-doubling route as the bifurcation parameter (a) is varied and exhibits chaotic dynamics for $a=3.7$. In order to stabilize the period-1 orbit (the fixed point of the map) embedded in the chaotic attractor, one can superimpose a linear delayed-feedback term onto the evolution equation of the map. The controlled system assumes the following form under the influence of superimposed feedback:

$$X_{n+1}=aX_n(1-X_n)+\gamma(X_n-X_{n-1}). \tag{2}$$

Figure 1(a) shows the influence of the linear delayed-feedback control for the control constant $\gamma=0.37$. The system dynamics converge on the period-1 orbit (fixed point of the map) upon implementation of the control. Subsequent to attainment of target state, the feedback term (control signal) vanishes as $X_n=X_{n-1}$. However, as the value of the control constant γ is decreased, a certain critical (minimum) value of $\gamma_c=0.35$ is found such that for $\gamma<\gamma_c$ control fails to stabilize the fixed point of the map. We define $\gamma<\gamma_c$ as the subthreshold region of control where, although suppression of chaotic dynamics is achieved, stabilization of the target period-1 orbit is unattainable. The evolution of the logistic map in the subthreshold control ($\gamma=0.32$) regime is shown in Fig. 1(b).

III. NOISE-AIDED CONTROL OF CHAOTIC DYNAMICS

Once the system is placed in the region of subthreshold control, external noise is added on the dynamics in the form

$$X_{n+1}=aX_n(1-X_n)+\gamma(X_n-X_{n-1})+\beta P_n, \tag{3}$$

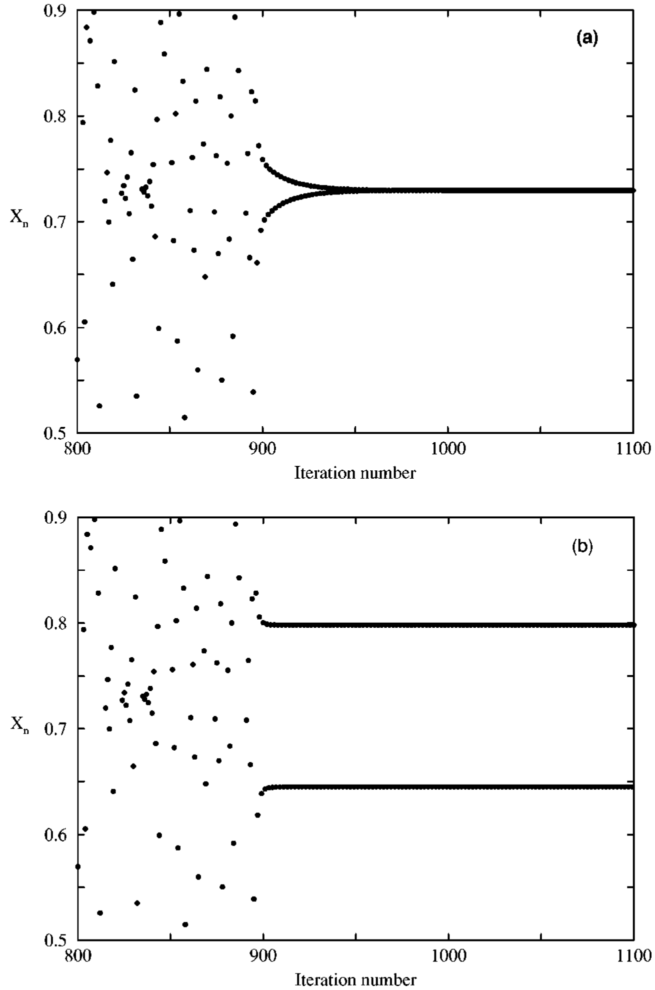

FIG. 1. (a) Control of chaotic dynamics for the logistic map using a linear delayed-feedback strategy. The value of the control constant used ($\gamma > \gamma_c$) is $\gamma = 0.37$. (b) Evolution of the map (with control) for $\gamma < \gamma_c (\gamma = 0.32)$. Although suppression of complexity is attained, the target period-1 orbit is elusive. This is the region of subthreshold control where external noise is superimposed.

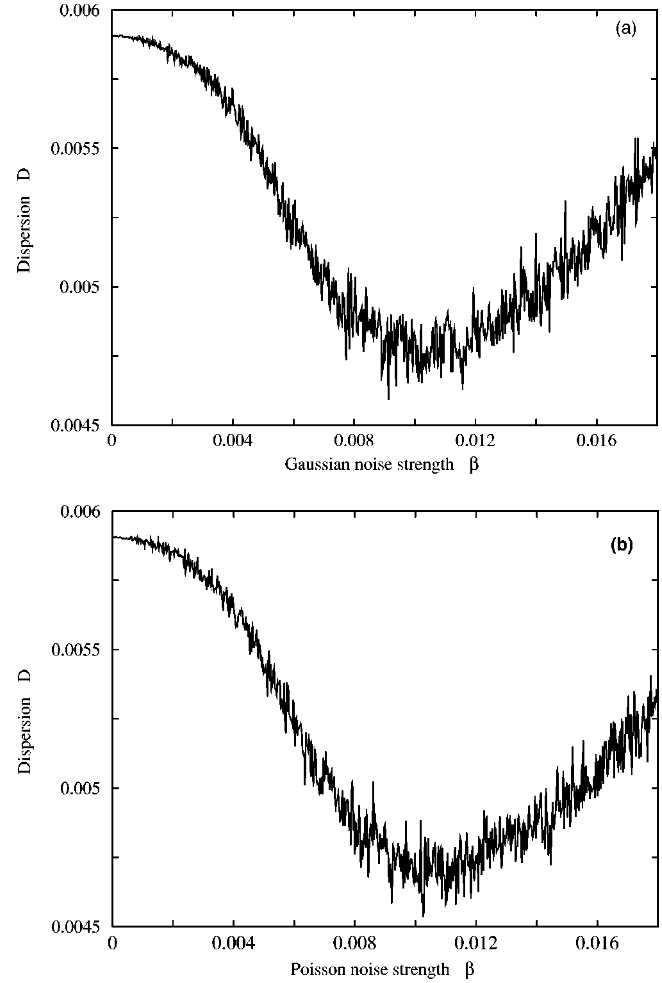

FIG. 2. (a) The dispersion curve (D) obtained as a function of the noise strength (β). The external noise used has a Gaussian distribution with a zero mean. The underlying resonance phenomena is manifested by the minima of the curve corresponding to the optimum noise level. The control constant (γ) in the subthreshold regime is fixed at $\gamma = 0.32$. (b) The dispersion curve (D) obtained as a function of the noise strength (β). The external noise used has a Poisson distribution with zero mean. The underlying resonance phenomena is manifested by the minima of the curve corresponding to the optimum noise level. The control constant (γ) in the subthreshold regime is fixed at $\gamma = 0.32$.

where P_n is a random variable (Gaussian and/or Poisson) with zero mean and standard deviation σ, and β is the amplitude of the variability (noise strength) superimposed on the logistic map. Augmenting β in small intervals, the response of the model system (with control) as a function of amplitude (β) of the additive noise is studied. In order to quantify the system dynamics in the presence of external noise a dispersion function (D) is defined, where

$$D = \frac{\sum_{n=n_i}^{n_f} |X_n - X_f|^2}{n_f - n_i} \tag{4}$$

measures the deviation of the controlled dynamics from the fixed point X_f of the map (target period-1 orbit).

Figure 2(a) shows the dependence of dispersion function (D) on the noise strength β. It exhibits a resonance behavior manifested by the presence of a distinct minima in the dispersion curve. This minima corresponds to an optimum level of noise for which the convergence of the system dynamics to the target period-1 orbit is maximum. This resonance phenomenon is qualitatively independent to the choice of seed used for generating the random variable. Moreover the minima in the dispersion curve persists for a string of random variables with a Poisson distribution (with zero mean), as shown in Fig. 2(b).

Another significant effect of noise on the controlled dynamics seems to be the increased speed at which the target state could be attained. Figure 3 shows the controlled logistic map without noise, and for the second curve noise is superimposed until the fixed point is attained (iteration number 1200), and subsequently turned off. The rate at which stabilization is achieved on the period-1 orbit is clearly faster in the presence of an optimal level of noise. However, we were not able to find any resonance effect (time to attain control vs β) for this aspect of the control. To reiterate, for the result of Fig. 3, noise is turned off subsequent to arriving at the fixed point X_f. It was realized that the convergence time decreases

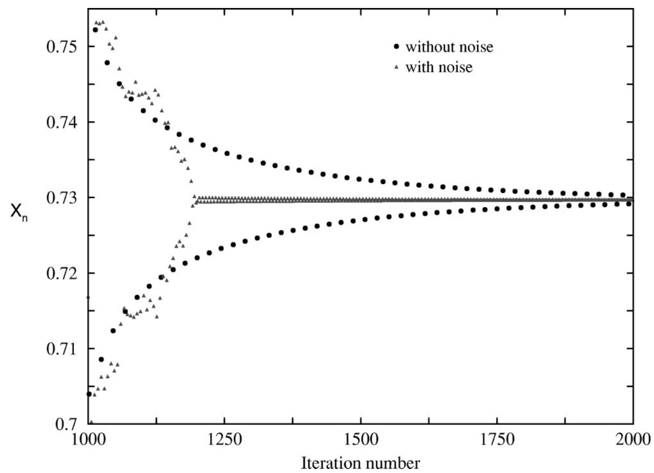

FIG. 3. The enhanced speed of control in the presence of an optimum level of noise ($\beta = 4.3 \times 10^{-4}$). The target period-1 orbit is attained 500 iteration units earlier in the presence of external noise. The control constant (γ) in the regime ($\gamma > \gamma_c$) is fixed at $\gamma = 0.351$.

as the level of noise was slowly increased, attaining a minima subsequent to the smooth convergence of the dynamics to the target state was hampered by increased level of noise. This faster convergence is a manifestation of the destruction of transients in the presence of noise. Therefore, it is valid (hence useful) for the values of control parameters where the controlled dynamics exhibit slowly decaying transient oscillations before converging on to the target period-1 state.

IV. DISCUSSION

The numerical results indicate that superimposed noise can indeed play a constructive role in controlling chaotic dynamics. It is reasonable to envisage an experimental situation where the superimposed feedback constant (γ) could not be incremented indefinitely. In such scenarios it could be desirable to attempt stabilization of the target state by superimposing external noise. Another possible application for noise aided control could involve the increased speed of stabilization observed in the presence of external noise. In certain situations, if the stabilization of target states is delayed by virtue of long transients, we suggest using external noise for a quicker targeting of desired dynamics.

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